CS3243 Tutorial 9

Eric Han

Nov 1, 2022

Annoucements

1. Assignment 7 scores are now on Luminus.
2. Highly recommended to send in your teaching feedback - I appreciate it greatly!
3. This tutorial is the last 'content' tutorial; next week is a recap tutorial.

Student Feedback on Teaching (SFT)

Feedback is *optional* but *highly encouraged*, access here: **https://es.nus.edu.sg/blue/**

- [**Tutorial Feedback**] Your feedback is important to me, and will be used to improve my teaching.
  - If I have helped your learning in any way, your positive feedback will be an encouragement to me.
  - If you find your learning can be enhanced by some action on my part, that feedback will be used to improve my teaching.
- [**Module Feedback**] Your feedback will be used to improve the module.
- Feedback is confidential to the university and anonymous to us.
- Avoid mixing the feedback; ie. project feedback to tutorial feedback.

Past student feedback had been used to improve teaching; ie. Telegram access to provide faster feedback.

Previously from T08, Q1

Consider below, a Vertex Cover where it is a set of vertices that covers all edges.

i. Write down the constraints as logical statements for a vertex cover of size 1.
ii. Apply the resolution algorithm in order to prove that the vertex 1 must be part of the vertex cover.

Recap

- What is a Vertex Cover of size k?
- How to formulate a KB problem?

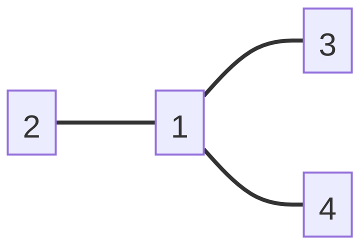

Figure 1: Graph for Vertex Cover CSP

Answer T8.Q1.i - Formulating KB

Variables:

- x_i represents i node on the graph that is in the vertex cover.

- $\neg x_i$ represents not in the vertex cover.

Constraints:

- Edge cover constraints:
  - $x_1 \vee x_2$;
  - $x_1 \vee x_3$;
  - $x_1 \vee x_4$
- Size $k = 1$ constraints: (We ignore contrapositive)
  - x_1 set then... $x_1 \implies \neg x_2$; $x_1 \implies \neg x_3$; $x_1 \implies \neg x_4$
  - x_2 set then... $x_2 \implies \neg x_3$; $x_2 \implies \neg x_4$;
  - x_3 set then... $x_3 \implies \neg x_4$

Then convert to CNF!

Answer T8.Q1.ii

Show $KB \vDash \alpha = x_1$; we resolve $KB \wedge \neg x_1$

1. $\neg x_1 \oplus x_1 \vee x_2 \implies x_2$
2. $x_2 \oplus \neg x_2 \vee \neg x_3 \implies \neg x_3$
3. $\neg x_1 \oplus x_1 \vee x_3 \implies x_3$
4. $x_3 \oplus \neg x_3 \implies \square$

Question 1

Having both good grades (G) and good communication skills (C) will increase your chances of performing well in your interview (I).

Table 1: Probability of I; $Pr[G = 1] = 0.7, Pr[C = 1] = 0.2$

G	C	$Pr[I = 1 \mid G, C]$
1	1	0.9
1	0	0.5
0	1	0.5
0	0	0.1

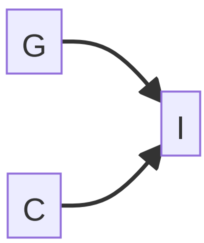

Figure 2: Belief Network

Recap

- How to read a Bayes Network?
- What is Marginalisation?

Answer

What is the probability that

a. Alice, who has poor grades and communication skills, performs well on her interview? - $Pr[I=1|G=0, C=0]=0.1$
b. Bob is a student with great communication skills, assuming we do not know anything about him? - $Pr[C=1]=0.2$
c. A student has good communication skills, given that he or she has performed well in an interview? - $Pr[C=1|I=1]=\frac{Pr[C=1,I=1]}{Pr[I=1]}=\frac{\sum_g Pr[G=g,C=1,I=1]}{\sum_{g,c} Pr[G=g,C=c,I=1]}=0.339$
    a. Are good communication skills independent of good performance in an interview? - Remember independence; $Pr[C=1|I=1]\neq Pr[C=1]=0.2$.

Question 2

Assume that 2% of the population in a country carry a particular virus (Y is a carrier). A test kit developed to detect the presence (X is positive test).

Y	$Pr[X=1\|Y]$
1	0.998
0	$1-0.996=0.004$

Recap

- What is Conditional Probability?
- What is Conditional Independence?

. . .

Conditional independence is a situation when an observation is redundant, i.e. $Pr[A|B,C]=Pr[A|C]$. Careful with the conditional independence here...

Answer 2a

Given that a patient is tested to be positive using this kit, what is the posterior belief that he is not a carrier?

$Pr[Y=0|X=1]=\frac{Pr[Y=0,X=1]}{Pr[X=1]}=\frac{Pr[Y=0,X=1]}{\sum_y Pr[Y=y,X=1]}=\frac{Pr[X=1|Y=0]\times Pr[Y=0]}{\sum_y Pr[Y=y,X=1]}=0.164$

Answer 2b

Patient tested positive again using the second kit (X_2 is the second test, X_1 is the first test). Assume conditional independence between results of different test kits given the patient's state of virus contraction.

$Pr[Y=0|X_1=1,X_2=1]=0.0008$

- $=\frac{Pr[Y=0,X_2=1,X_1=1]}{Pr[X_1=1,X_2=1]}=\frac{Pr[X_2=1,X_1=1|Y=0]Pr[Y=0]}{Pr[X_1=1,X_2=1]}$ (Conditional Probability)
- $=\frac{Pr[X_2=1|Y=0]Pr[X_1=1|Y=0]Pr[Y=0]}{Pr[X_1=1,X_2=1]}$ (Conditional Independence)
- $=\frac{Pr[X_2=1|Y=0]Pr[X_1=1|Y=0]Pr[Y=0]}{\sum_y Pr[X_1=1,X_2=1,Y=y]}$ (Marginalisation)
- $=\frac{Pr[X_2=1|Y=0]Pr[X_1=1|Y=0]Pr[Y=0]}{\sum_y Pr[X_1=1|Y=y]Pr[X_2=1|Y=y]Pr[Y=y]}$ (Conditional Probability, Independence)

Very interesting, read more...

- Can you solve the false positive riddle?: https://youtu.be/1csFTDXXULY
- False-positive paradox: https://en.wikipedia.org/wiki/Base_rate_fallacy
- Prior, Posterior...

Question 3

Construct a Bayesian network and determine the probability

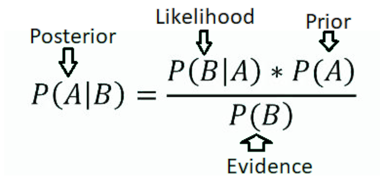

Figure 3: Prior vs Posterior

$$Pr[WG = 1, RS = 1, R = 0, S = 0]$$

Recap

- Any useless variables from the table?

Answer

Lemma 1. Given two random boolean variables A and B, if $Pr[A|B] = 0$ and $Pr[A|\neg B] = 1$ then $Pr[A] = 1 - Pr[B]$; in fact, $A \equiv \neg B$.

Not difficult to proof (*Note the notation change.*):

$$Pr[A|B] = 0 \land Pr[A|\neg B] = 1 \implies Pr[A] = 1 - Pr[B]$$

$Pr[A]$

- $= Pr[A \land B] + Pr[A \land \neg B]$ (Marginalisation)
- $= Pr[A|B] \times Pr[B] + Pr[A|\neg B] \times Pr[\neg B]$ (Conditional Probability)
- $= Pr[\neg B] = 1 - Pr[B]$ (Subt given)

So, $Pr[A \land \neg B \land \cdots] = Pr[\cdots|A \land \neg B] \times Pr[A \land \neg B] = Pr[\cdots|A] \times Pr[A]$,

- Since, $Pr[A \land \neg B] = Pr[A|\neg B] \times Pr[\neg B] = Pr[A]$;
- And, $Pr[\cdots|A \land \neg B] = Pr[\cdots \land \neg B|A]/Pr[\neg B|A] = Pr[\cdots|A]$

Since $S \equiv \neg RS$, so:

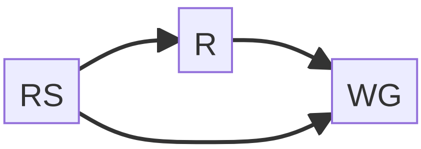

Figure 4: RS, WG, R network.

$Pr[RS = 1] = 0.7$

RS	$Pr[R = 1|RS]$
1	0.9
0	0.1

RS	R	$Pr[WG = 1|RS, R]$
1	1	0.8
1	0	0.1
0	1	0.95
0	0	0.9

$Pr[WG = 1, RS = 1, R = 0, S = 0] = Pr[WG = 1, RS = 1, R = 0]$

- $= Pr[WG = 1, R = 0|RS = 1] \times Pr[RS = 1]$ (Conditional Probability)